

# The Skeleton in the Closet: Rethinking Curriculum Maps

Welcome to Jenks

# Who is this person?

**Kate Nowak**

katherine.nowak@gmail.com

@k8nowak

Favorite Standard: A-REI.10

Least Favorite Standard: G-CO.10

## At your school or district how are they adapting CCSS to a new curriculum?



Adopted a new published curriculum



Trying to adapt existing curriculum

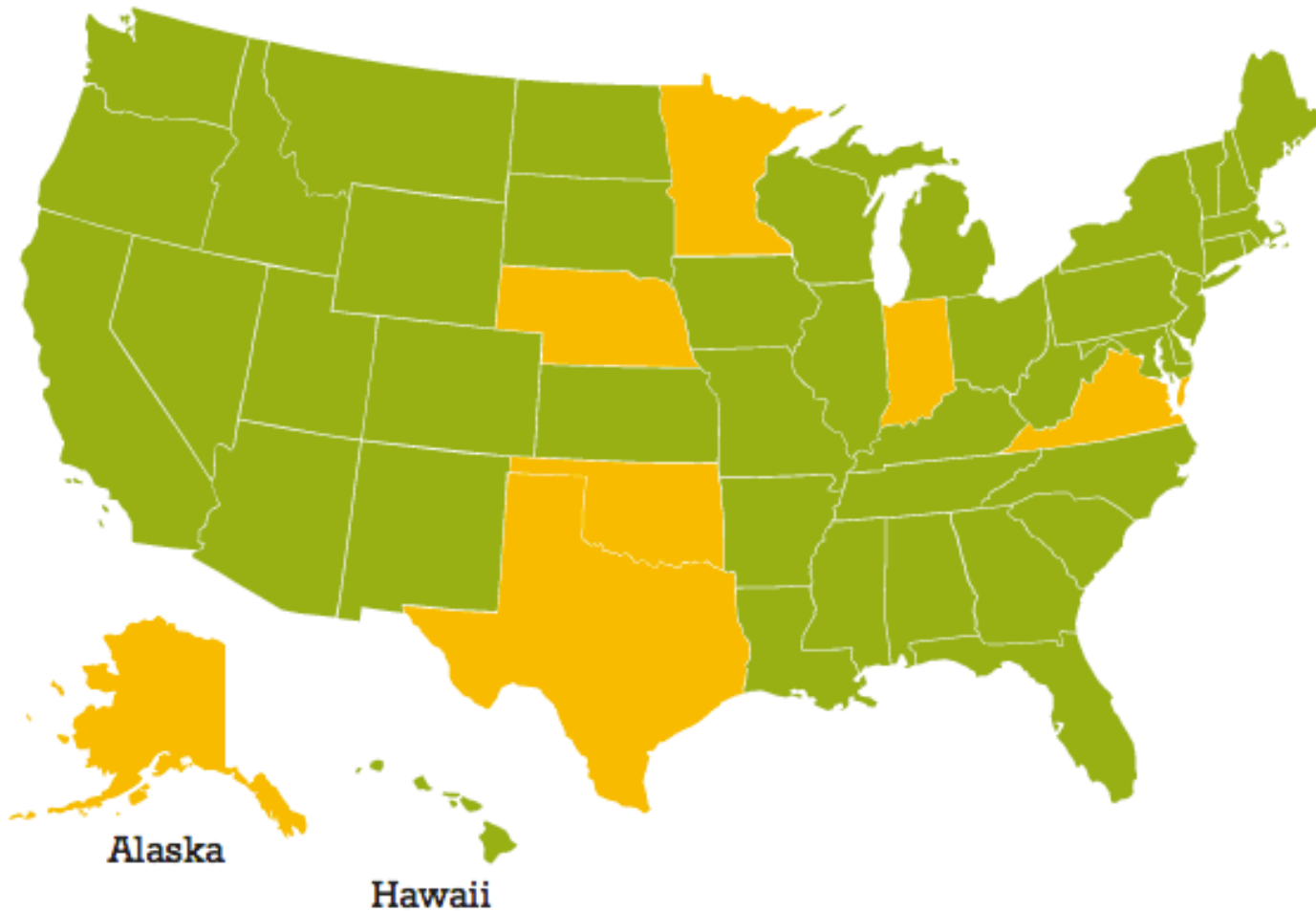


District-wide level work to create materials



Passed out a copy of the CCSS and told “good luck!”

# A challenge for schools, states, and publishers

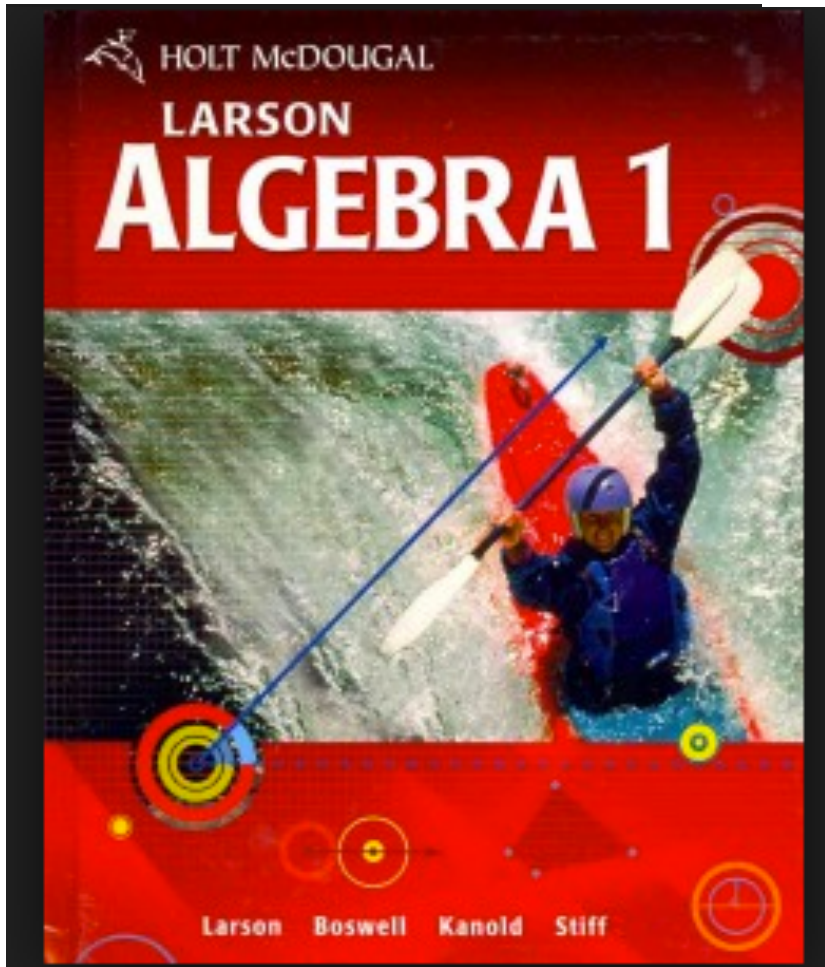


# A Common Common Core Misconception

*“Just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.”*

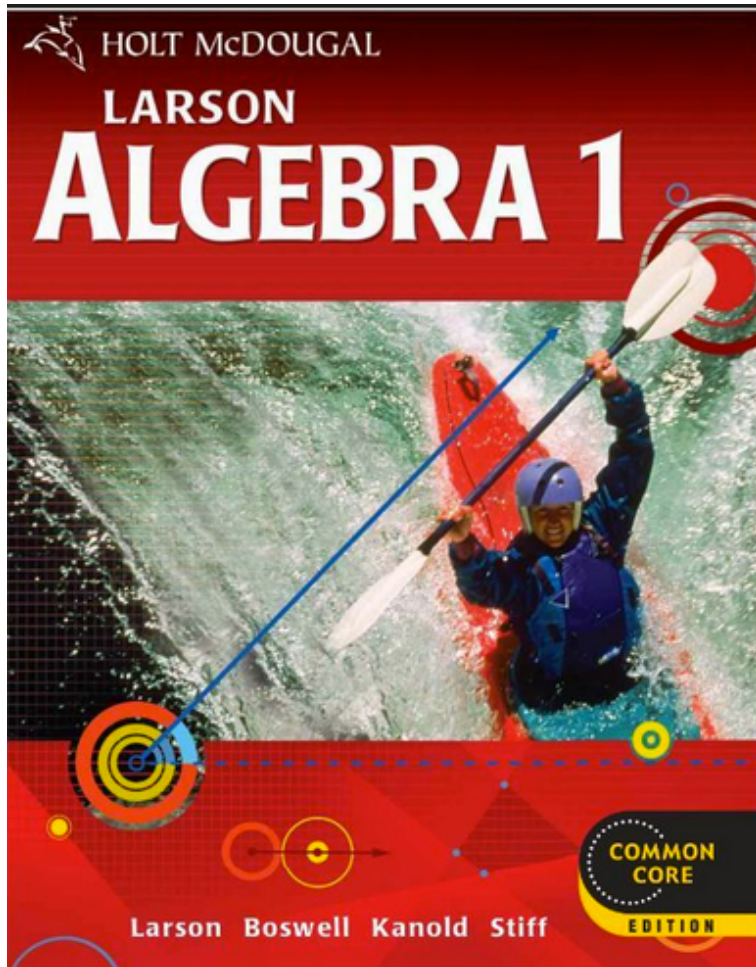
# Crosswalks are not the answer





Not Common Core  
aligned!






# New “CCSS Aligned” Textbooks

## 5.2 Evaluate and Graph Polynomial Functions

**Before** You evaluated and graphed linear and quadratic functions.  
**Now** You will evaluate and graph other polynomial functions.  
**Why?** So you can model skateboarding participation, as in Ex. 55.



**Key Vocabulary**

- polynomial
- polynomial function
- synthetic substitution
- end behavior

Recall that a monomial is a number, a variable, or a product of numbers and variables. A **polynomial** is a monomial or a sum of monomials. A **polynomial function** is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where  $a_n \neq 0$ , the exponents are all whole numbers, and the coefficients are all real numbers. For this function,  $a_n$  is the **leading coefficient**,  $n$  is the **degree**, and  $a_0$  is the **constant term**. A polynomial function is in **standard form** if its terms are written in descending order of exponents from left to right.

Common Polynomial Functions			
Degree	Type	Standard form	Example
0	Constant	$f(x) = a_0$	$f(x) = -14$
1	Linear	$f(x) = a_1 x + a_0$	$f(x) = 5x - 7$
2	Quadratic	$f(x) = a_2 x^2 + a_1 x + a_0$	$f(x) = 2x^2 + x - 9$
3	Cubic	$f(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$	$f(x) = x^3 - x^2 + 3x$
4	Quartic	$f(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$	$f(x) = x^4 + 2x - 1$

**EXAMPLE 1 Identify polynomial functions**

Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

a.  $h(x) = x^4 - \frac{1}{4}x^2 + 3$       b.  $g(x) = 7x - \sqrt{3} + \pi x^2$   
c.  $f(x) = 5x^2 + 3x^{-1} - x$       d.  $k(x) = x + 2^4 - 0.6x^5$


**Solution**

a. The function is a polynomial function that is already written in standard form. It has degree 4 (quartic) and a leading coefficient of 1.  
b. The function is a polynomial function written as  $g(x) = \pi x^2 + 7x - \sqrt{3}$  in standard form. It has degree 2 (quadratic) and a leading coefficient of  $\pi$ .  
c. The function is not a polynomial function because the term  $3x^{-1}$  has an exponent that is not a whole number.  
d. The function is not a polynomial function because the term  $2^4$  does not have a variable associated with it, so it is not a term of the polynomial function.

(LARSON, 2011) 5.2 Evaluate and Graph Polynomial Functions 337

## 2.2 Evaluate and Graph Polynomial Functions

**Before** You evaluated and graphed linear and quadratic functions.  
**Now** You will evaluate and graph other polynomial functions.  
**Why?** So you can model skateboarding participation, as in Ex. 55.



**Key Vocabulary**

- polynomial
- polynomial function
- synthetic substitution
- end behavior

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(LARSON, 2012) 2.2 Evaluate and Graph Polynomial Functions 95

# New "CCSS Aligned" Textbooks

**B Apply**

**43. Think About a Plan** Suppose you have a part-time job delivering packages. Your employer pays you a flat rate of \$9.50 per hour. You discover that a competitor pays employees \$2 per hour plus \$3 per delivery. How many deliveries would the competitor's employees have to make in four hours to earn the same pay you earn in a four-hour shift?

- How can you write a system of equations to model this situation?
- Which method should you use to solve the system?
- How can you interpret the solution in the context of the problem?

Solve each system.

44.  $\begin{cases} 5x + y = 0 \\ 5x + 2y = 30 \end{cases}$

45.  $\begin{cases} 2m = -4n - 4 \\ 3m + 5n = -3 \end{cases}$

46.  $\begin{cases} 7x + 2y = -8 \\ 8y = 4x \end{cases}$

47.  $\begin{cases} 2m + 4n = 10 \\ 3m + 5n = 11 \end{cases}$

48.  $\begin{cases} -6 = 3x - 6y \\ 4x = 4 + 5y \end{cases}$

49.  $\begin{cases} \frac{x}{3} + \frac{4y}{3} = 300 \\ 3x - 4y = 300 \end{cases}$

50.  $\begin{cases} 0.02a - 1.5b = 4 \\ 0.5b - 0.02a = 1.8 \end{cases}$

51.  $\begin{cases} 4y = 2x \\ 2x + y = \frac{x}{2} + 1 \end{cases}$

52.  $\begin{cases} \frac{1}{4}x + \frac{2}{3}y = 1 \\ \frac{1}{4}x - \frac{1}{3}y = 2 \end{cases}$

**53. Error Analysis** Identify and correct the error shown in finding the solution of  $\begin{cases} 3x - 4y = 14 \\ x + y = -7 \end{cases}$  using substitution.

$$\begin{array}{l} x + y = -7 \\ y = -7 - x \\ 3x - 4y = 14 \\ 3x - 4(-7 - x) = 14 \\ 3x - 28 - 4x = 14 \\ -x - 28 = 14 \\ -x = -42 \\ x = -42 \\ y = -7 - (-42) \\ y = 35 \end{array}$$

**54. Break-Even Point** Jenny's Bakery sells carrot muffins at \$2 each. The electricity to run the oven is \$120 per day and the cost of making one carrot muffin is \$1.40. How many muffins need to be sold each day to break even?

**55. Open-Ended** Write a system of equations in which both equations must be multiplied by a number other than 1 or -1 before using elimination. Solve the system.

**56. Chemistry** A scientist wants to make 6 milliliters of a 30% sulfuric acid solution. The solution is to be made from a combination of a 20% sulfuric acid solution and a 50% sulfuric acid solution. How many milliliters of each solution must be combined to make the 30% solution?

**57. Writing** Explain how you decide whether to use substitution or elimination to solve a system.

**58.** The equation  $3x - 4y = 2$  and which equation below form a system with no solutions?

(A)  $2y = 1.5x - 2$

(C)  $3x + 4y = 2$

(B)  $2y = 1.5x - 1$

(D)  $4y - 3x = -2$

For each system, choose the method of solving that seems easier to use. Explain why you made each choice. Solve each system.

$\begin{cases} 3x - y = 5 \\ 2x + 3y = 1 \end{cases}$

$\begin{cases} 2x - 3y = -4 \\ 2x + 5y = 1 \end{cases}$

61.  $\begin{cases} 6x - 3y = 3 \\ 5x - 5y = 10 \end{cases}$

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(PEARSON 2011)

(PEARSON 2012)

# New expectations for students

Getting (or guessing) the right answers is no longer sufficient.

Students are expected to be able to **use** and **apply** their mathematical knowledge.

There are the Standards for Mathematical Practice.

# CCSS Mathematical Practices

## OVERARCHING HABITS OF MIND

1. Make sense of problems and persevere in solving them
6. Attend to precision

## REASONING AND EXPLAINING

2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others

## MODELING AND USING TOOLS

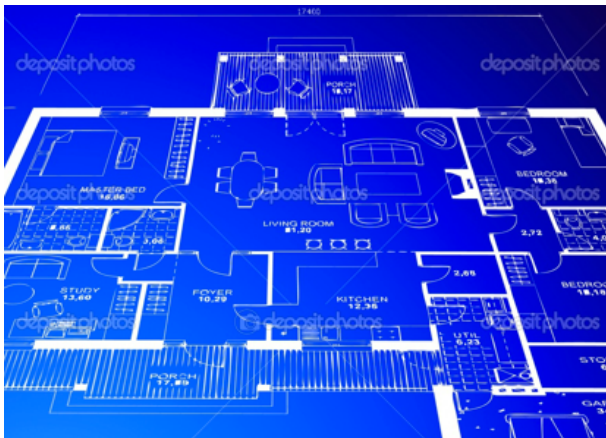
4. Model with mathematics
5. Use appropriate tools strategically

## SEEING STRUCTURE AND GENERALIZING

7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

# Discussion

What makes a good unit?



# Unit Blueprints

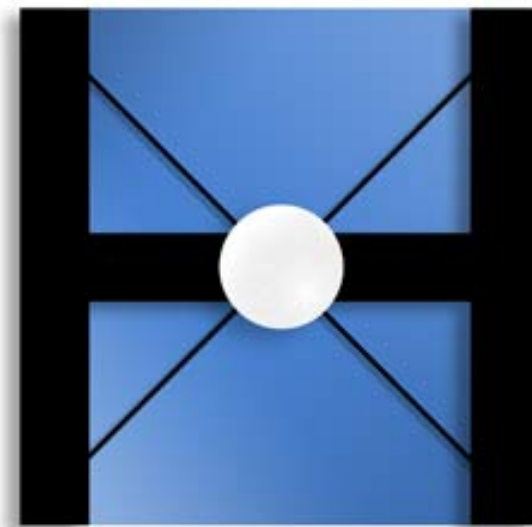
- D. Represent and solve equations and inequalities graphically.  
illustrations

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). illustrations

# Multiple Purposes

- Supporting teacher's efforts to implement the Common Core with fidelity.
- Undergirding efforts to create new curricula that are mathematically and pedagogically coherent.
- Providing a basis for focused professional development for teachers.
- Informing rubrics for reviewing and certifying existing resources, including educational technologies.
- Supporting districts in developing Common Core implementation plans.
- Supporting publishers in structuring their materials

# Course Plans and Unit Blueprints



HIGH TECH HIGH

With support from the

THE WILLIAM AND FLORA  
**HEWLETT**  
FOUNDATION

HTH



Amy Callahan

Sarah Strong

Jade White

# Have you reviewed/selected a curriculum



Yes, at least once



No, this happens, but I've not participated



No, we have written our own homebrew curriculum



No, a complete curriculum is prescribed by higher authority

# Design criteria for selecting and sequencing activities within a Unit

- **What** is the task/lesson/project/activity and its purpose?
- **Where** does it fit within the sequence of the unit?
- **How** does it accomplish its purpose (including instructional strategies)?
- **Why** is this coherent mathematically and pedagogically? (i.e. what is the rationale)

# Traditional

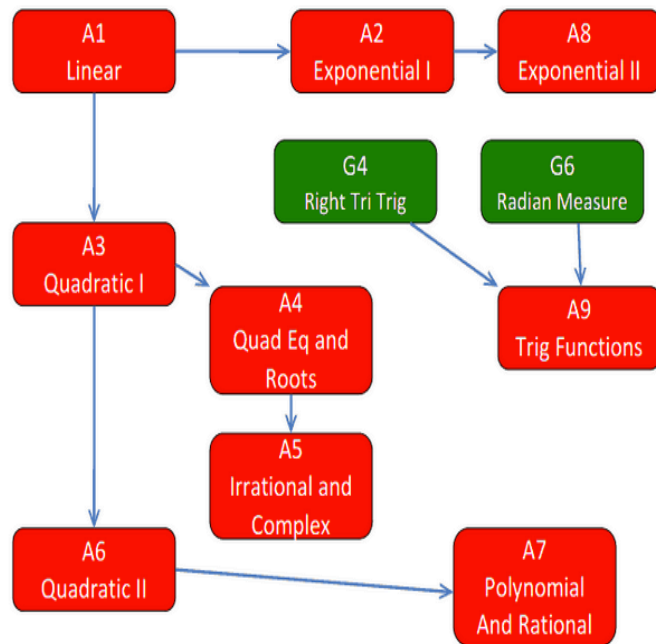
Algebra 1	Geometry	Algebra 2
S1 Univariate Stats	G1 Rigid Transformations	A5 Irrational & Complex Numbers
A1 Linear Equations and Inequalities, including systems	G2 Congruence Theorems	A6 Exponents 2
S2 Bivariate Statistics	G3 Similarity	A7 The Beastiary
F1 Bring da Functions	G4 Right Triangle Trig	A8 Periodic Functions
A2 Exponential Functions 1	G5 Measurement	S3 Probability
A3 Quadratic Functions 1	G6 Circles	S4 Statistics 2
A4 Quadratic Equations and Roots	G7 Capstone Modeling with Geometry	

# Integrated

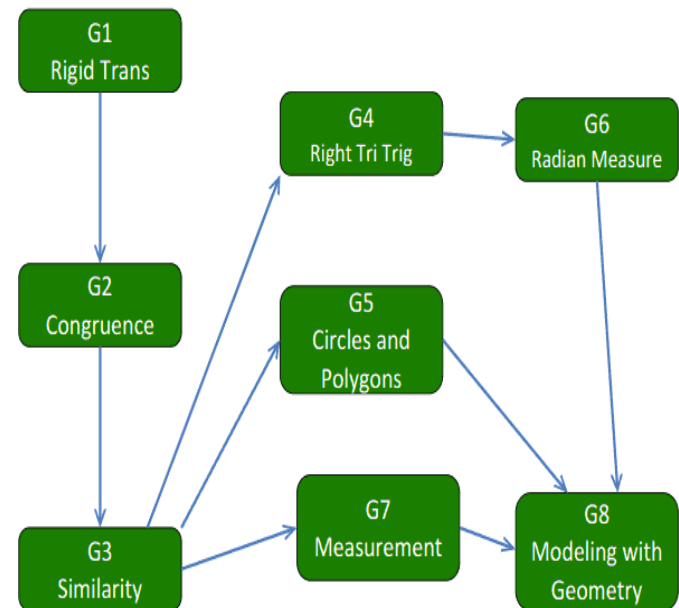
Integrated 1	Integrated 2	Integrated 3
S1 Univariate Stats	S3 Probability	A6 Exponents 2
S2 Bivariate Statistics	A3 Quadratic Functions 1	A7 The Beastiary
A1 Linear Equations and Inequalities, including systems	A4 Quadratic Equations and Roots	G5 Measurement
F1 Bring da Functions	A5 Irrational & Complex Numbers	A10 Trigonometric Functions
A2 Exponential Functions 1	G3 Similarity	S4 Statistics 2
G1 Rigid Transformations	G4 Right Triangle Trig	G6 Circles
G2 Congruence Theorems		G7 Capstone Modeling with Geometry

# Flexible Ordering with Dependencies

Unit Dependencies (Algebra)



Unit Dependencies (Geometry)



# Before we look at a blueprint...

## **A Unit Blueprint includes...**

- A coherent mathematical and pedagogical flow
- Rationales behind choices made
- Links to suggested tasks and lessons
- Sample assessment items

## **A Unit Blueprint does not include...**

- Pedagogical support
- Lesson Plans
- A comprehensive set of assessments

<b>Unit</b>	<b>G4</b>
<b>Title</b>	<b>Right Triangle Trigonometry</b>
<b>Target Standards</b>	<b>G-SRT.6-8</b>
<b>Mathematical Goals</b>	<ul style="list-style-type: none"> <li>• By similarity, show that side ratios in right triangles are properties of the angles in the triangle in a trig table <b>G-SRT.6</b></li> <li>• Define the trigonometric ratios <b>G-SRT.6</b></li> <li>• Explain and use the relationship between sine and cosine in complementary angles <b>G-SRT.7</b></li> <li>• Use trigonometric ratios to solve a variety of modeling problems <b>G-SRT.8</b></li> </ul>
<b><i>The story before this unit (including prior knowledge)</i></b>	In previous geometry units, students have proven various theorems about triangles including the measures of interior angles of a triangle sum to $180^\circ$ and the base angles of isosceles triangles are congruent. Students have also established the AA criteria for similar triangles and have experience working with this theorem. Most recently, students have used properties of similar triangles (like proportionality of side lengths and congruence of corresponding angles) to solve problems.
<b><i>The part of the story happening in this unit</i></b>	In this unit, students will extend their understanding of similar triangles as they explore the side ratios of these triangles and look for patterns arising therein. They will build a table from their own exploration of side length ratios before more formally defining the ratios as the trigonometric ratios. They will use these ratios to solve a variety of real world problems. Students will also spend some time exploring the relationship between some of the trigonometric relationships and some of their special properties.
<b><i>The story after this unit</i></b>	In their future study of trigonometry, students will connect their knowledge of right triangle trigonometry to the unit circle. They will see that the sine and cosine value of an angle represent the height and width of a coordinate point on the circle formed by the terminal ray of the angle, and extend that understanding to angles outside quadrant 1. They will make sense of the oscillating values of sine and cosine, as well as the asymptotic behavior of tangent.

## UNIT FLOW SUMMARY

<b>UNIT G4 (10-12 days)</b>	<b>Right Triangle Trigonometry</b>
<b>Section 0 (1 day)</b>	<b>Diagnostic Pre-Unit Assessment</b>
<b>Section 1 (1 day)</b>	<b>Motivate the need for the trigonometric ratios</b>
<b>Section 2 (2 days)</b>	<b>Build a table of side ratios of similar triangle</b>
<b>Section 3 (1 day)</b>	<b>Define trigonometric ratios</b>
<b>Section 4 (1 day)</b>	<b>Discover relationship between sin/cos and complementary angles</b>
<b>Section 5 (3-5 day)</b>	<b>Solve problems using basic trig ratios</b>
<b>Section 6 (1 day)</b>	<b>Summative Assessment</b>

Section 5: 3-5 days	Solve problems using basic trig ratios
Mathematical Goals	<p>Students will:</p> <ul style="list-style-type: none"> <li>Use trig ratios to solve a variety of modeling problems G-SRT.8</li> </ul>
Narrative overview of section (and how the standards are achieved)	<p>In this section, students will bring together all of their knowledge of the trig ratios and solve real world modeling problems. G-SRT.8 This section includes various contexts that will likely engage students including a look at TV viewing angles and a clinometer activity where they go around school and find the height of tall objects. At the end of this section, there is a task that introduces inverse trig functions which students are not familiar with yet. This task can be used to necessitate and introduce the inverse of sin, cos, and tan. The thoroughness of the treatment of this topic is at the teachers' discretion. The tasks in this section are not placed in a particular order and any sampling of them would achieve the standard G-SRT.8.</p>
Sample Activity 5.1	<p><a href="#">Sofa Away from Me</a>, Mathalicious</p> <p><b>WHAT:</b> Given a television of a certain size, where's the best place to put the couch? This lesson uses right triangle trigonometry and a rational function to explore the percent of your visual field that is occupied by the area of a television. Students use right triangle trigonometry to find the visible width, height, and viewing area for various distances from the television. G-SRT.8 They will find and plot the percent of a person's field of view filled by a 60-in TV and write a rational function for the percent they find. MP8 Finally, they will solve, algebraically or by graphing, the function to find the distance where the TV fills 100% of your view</p> <p><b>WHY:</b> This problem provides an engaging and challenging modeling application of right triangle trigonometry. MP4, MP1 It also features a graph of a rational function when students graph "distance from screen" vs. "screen as a percent of viewing area" but this part is highly scaffolded and a previous understanding of rational functions is not necessary. The lesson provides a nice bridge between algebra and geometry topics.</p>
Sample Activity 5.2	<p><a href="#">Satellite</a>, Illustrative Mathematics</p> <p><b>WHAT:</b> This task is an example of applying geometric methods to solve design problems and satisfy physical constraints. This task models a satellite orbiting the earth in communication with two control stations located miles apart on earth's surface. Students determine distances between various locations and given a rate of travel, students determine the time it takes to travel between two specified locations. Part c here introduces the law of cosines and is beyond the scope of this section but the teacher could make the choice to include it here. G-SRT.8</p> <p><b>WHY:</b> As with the others in this section, this task provides an engaging modeling context to apply the trig ratios to solve problems. MP4</p>

# Details

The course plans and unit blueprints will be published at [illustrativemathematics.org](https://illustrativemathematics.org)



They will be freely available  
(under Creative Commons License)