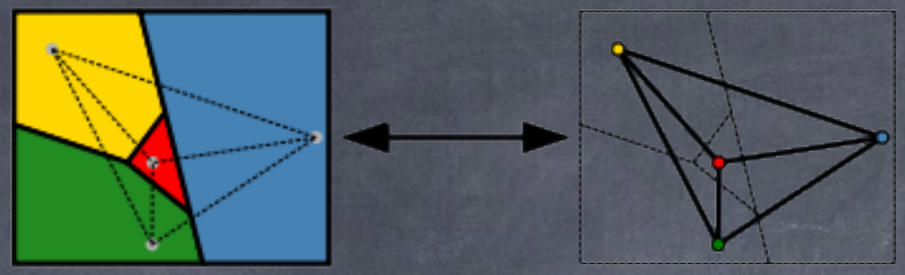

Young Mathematicians The Thrill of Mathematical Discovery

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What is Math?

Doing mathematics should always mean finding patterns and crafting beautiful and meaningful explanations."

-Paul Lockhart

Setting Students up for Mathematical Discoveries

- Easy to get started
- Open middle, and/or open ended
- Multiple levels of success/understanding
- Provides a mathematical sandbox that is rich, beautiful, elegant, etc.
- Inspires students to use mathematical practices to help understand and explain why
- Motivates students to speak with each other and collaborate mathematically

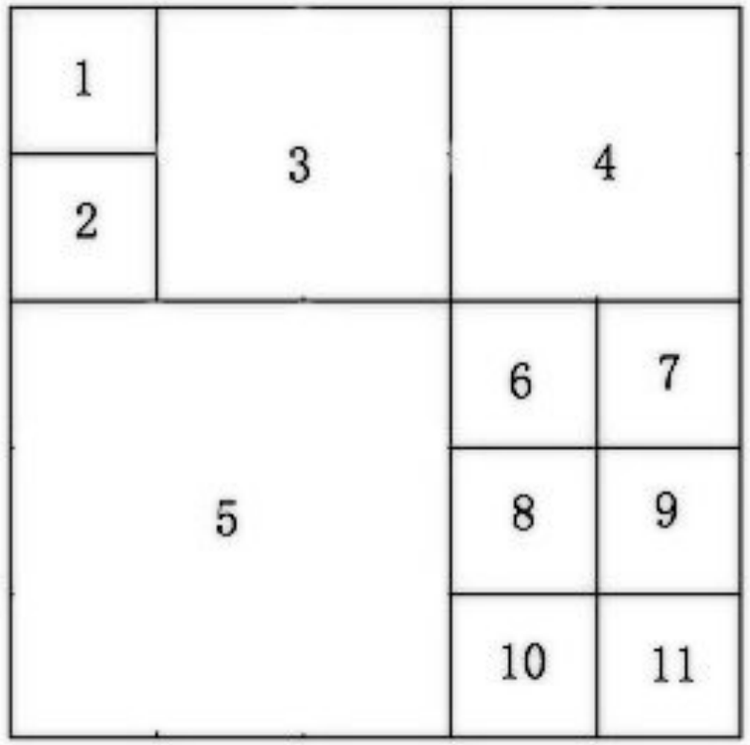
Mathematical Sandbox

- Perplexing context
- Data collection, organization, modeling, and patterns throughout
- Motivate students to be systematic in their research
- Encourage students to work collaboratively

The Squareable Puzzle

Call a number "squareable" if it's possible to build a square out of precisely that many squares. For example, 11 is squareable: 11 squares can be fit together to perfectly form another square.

(1st/2nd)

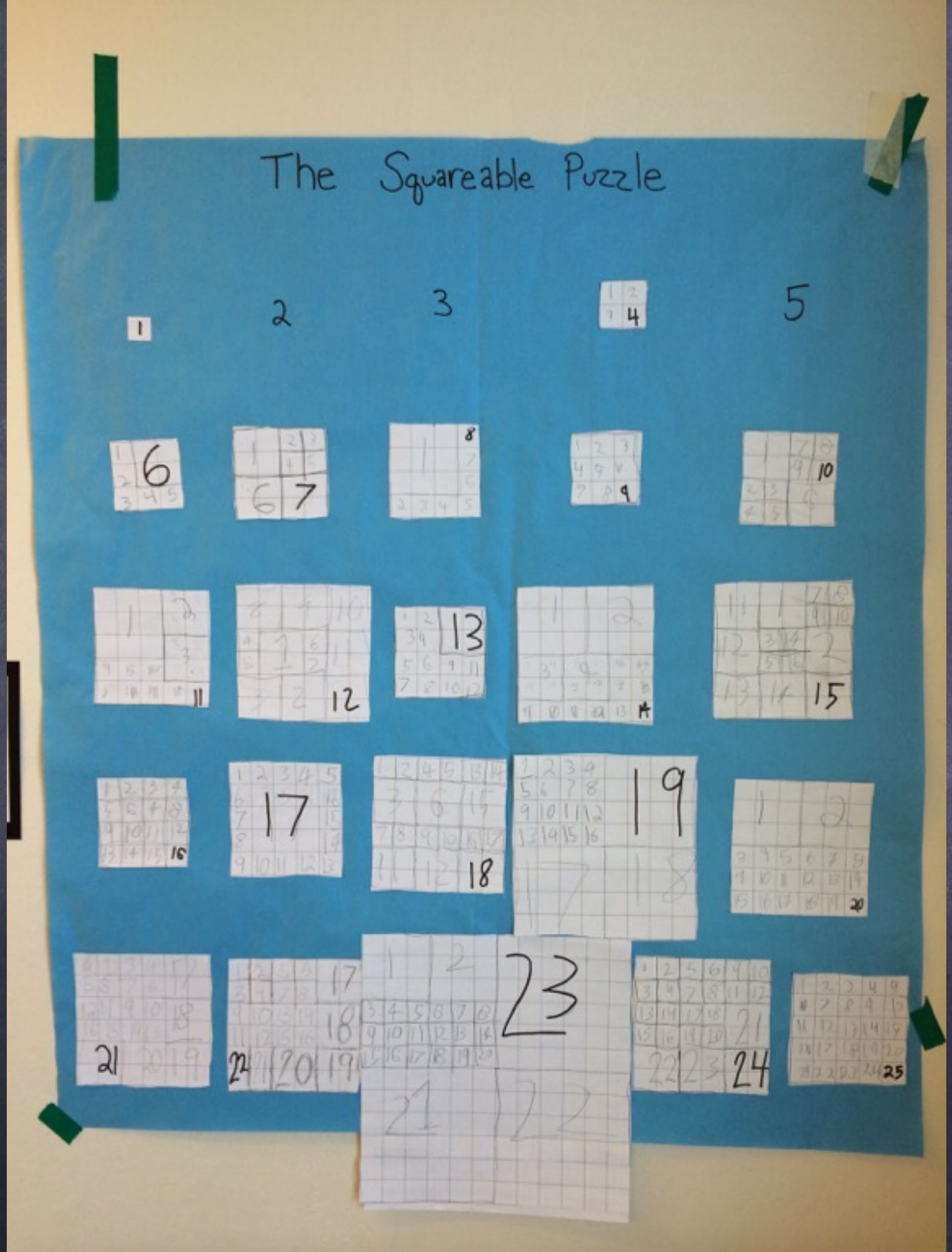


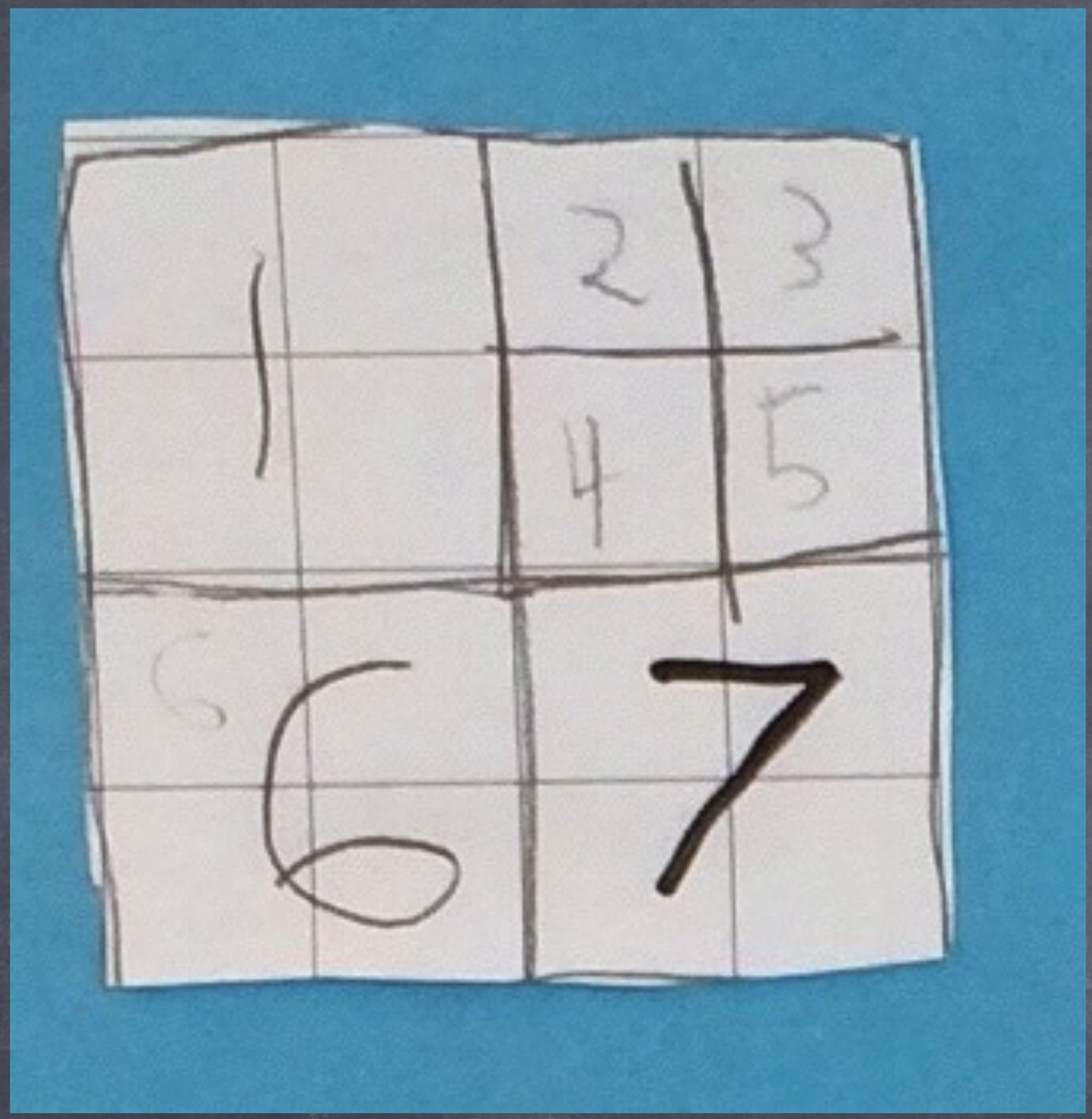
11 is squareable: 11 squares can be fit together to perfectly form another square. Daniel Finkel

The question is: can you find all the numbers less than 30 that are squareable? Is there a pattern? Can you predict squareability in general?

Dan Finkel's Puzzle in NY Times Number Play via Gary Antonik

The Squareable Puzzle

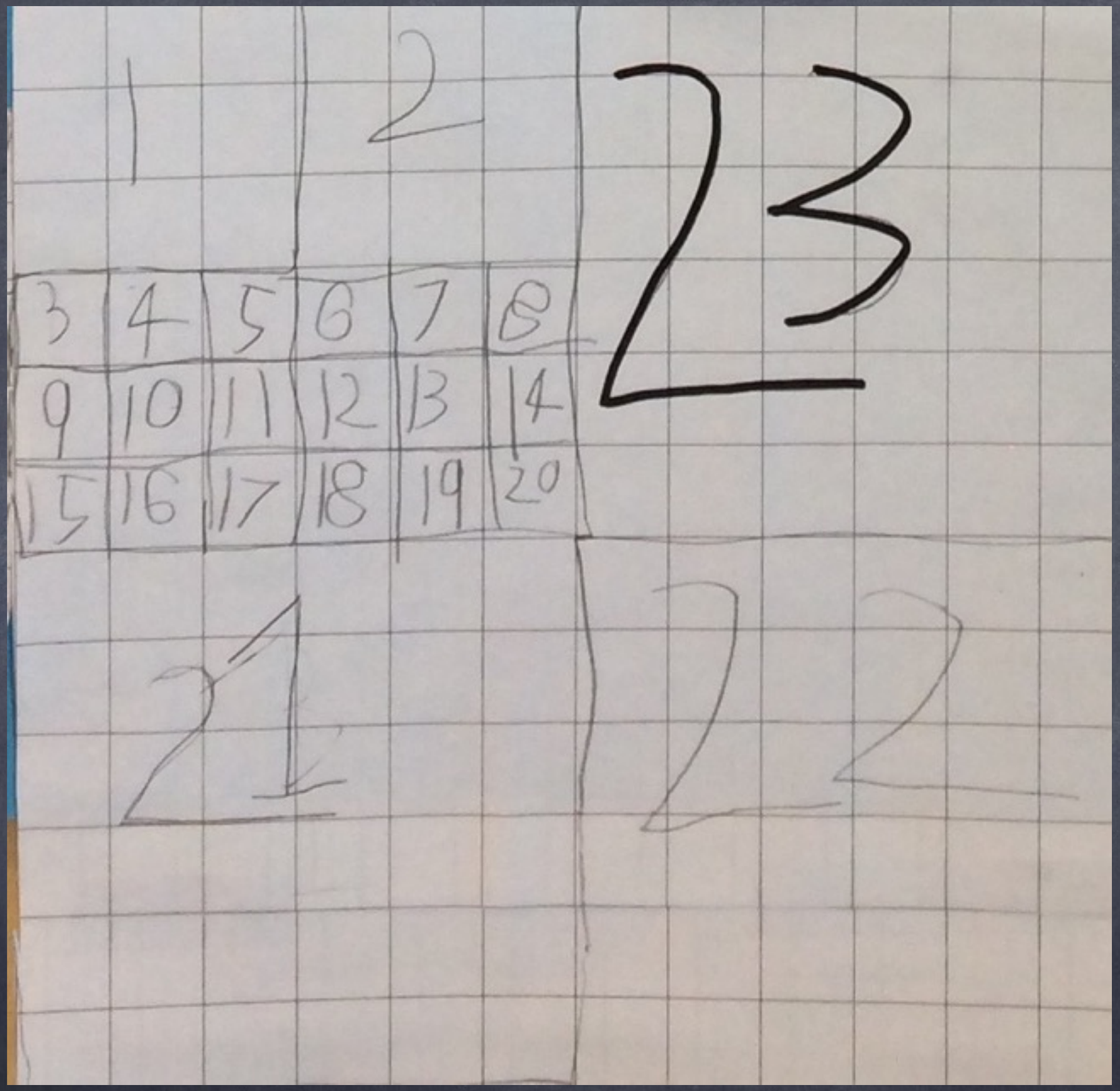




"Banana Split Operation"

4+3-1

🍌🍦



"Gigantor Operation"

20 + 3

Hundreds Game (K-2)

- Two players start from 0 and alternatively add a number from 1 to 10 to the sum
- The player who reaches 100 wins
- Part of a family of games called "NIM"

Hundreds Chart

NAME _____

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Subtraction Reversal Mysteries (1st-8th)

- Each player rolls a 10-sided die two times
- Find the difference between the largest number you can make using both numbers and the smallest number you can make. For example: (3,5): 53-35 = 18
- Whoever has the largest difference wins
- Play several rounds, Recording your results on paper

Megan's Subtraction Mystery Poster

0	9	18	27	36	45	54	63	72	81
(0,0)	(0,1)	(0,2)	(0,3)	(0,4)	(0,5)	(0,6)	(0,7)	(0,8)	(9,0)
(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)	(1,8)	(1,9)	
(2,2)	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)	(2,8)	(2,9)		
(3,3)	(3,4)	(3,5)	(3,6)	(3,7)	(3,8)	(3,9)			
(4,4)	(4,5)	(4,6)	(4,7)	(4,8)	(4,9)				
(5,5)	(5,6)	(5,7)	(5,8)	(5,9)					
(6,6)	(6,7)	(6,8)	(6,9)						
(7,7)	(7,8)	(7,9)							
(8,8)	(8,9)								
(9,9)									

You get two numbers. Make the biggest two-digit number and subtract the smallest number you can make out of those two numbers.

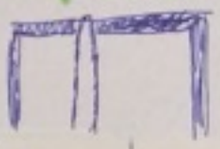
I observed: All of these answers are multiples of nine.

The pattern is: As the answers grew bigger, the gaps in the pairs grew bigger too. That's because the gaps grew according to how many nines there were in the answer.

Examples:

97	82	87	51
-79	-28	-78	-15
18	54	09	36

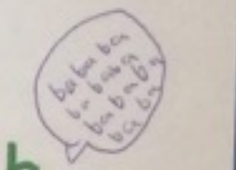
Subtraction Reversal Mysteries

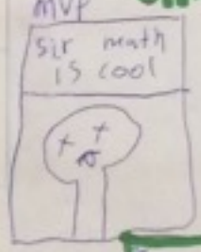


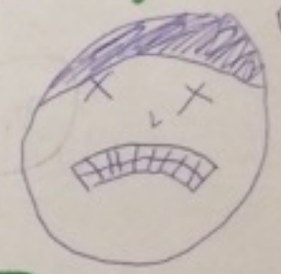
By: Devi and Sam

0	9	18	27	36	45	54	63	72	81
0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9
1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	
2,2	2,3	2,4	2,5	2,6	2,7	2,8	2,9		
3,3	3,4	3,5	3,6	3,7	3,8	3,9			
4,4	4,5	4,6	4,7	4,8	4,9				
5,5	5,6	5,7	5,8	5,9					
6,6	6,7	6,8	6,9						
7,7	7,8	7,9							
8,8	8,9								
9,9									

If you roll a and b, then the difference you multiply by 9 and you get your answer.







$(a-b)\times 9$

a = big number

b = small number

Number Bracelets (3rd-6th)

- First player rolls twice, writing the two number down on a piece of inch grid paper.
- The second player does the same.
- Then students add their two numbers together, but discard the tens-place, only writing the ones-place in the next number.
- Players continue this process until they get back to the two numbers they started with (in the same order).

8 8 6 4 0 4 4 8 2 0

2 2 4 6 0 6 6 2 8 0

8 8 ✓

0 4 4 8 2 0 2 2 4 6

0 6 6 2 8 0 8 8 6 4

0 4 ✓

Sizes of a Number Bracelet

Small

9	7	6	3	9	2	1	3	4	7
1	8	9	7						

Other Small Combinations:

(0,5)=3 (4,7)=12 (9,2)=12.

Small combinations range from 3 to 12, but you don't count the last two numbers.

Medium

8	6	4	0	4	4	8	2	0	2
2	4	6	0	6	6	2	8	0	8

8	6

Other Medium Combinations:

(3,7)=18 (4,4)=20 (4,2)=20

Large

5	6	1	7	8	5	3	8	1	9
0	9	9	8	7	5	2	7	9	6
5	1	6	7	3	0	3	3	6	9
5	4	9	3	2	5	7	2	9	1
0	1	1	2	3	5	8	3	1	4
5	9	4	3	7	0	7	7	4	1

Other Large Combinations:

(4,5)=60 (3,5)=60

What is the longest number bracelet? How Can you be Sure?

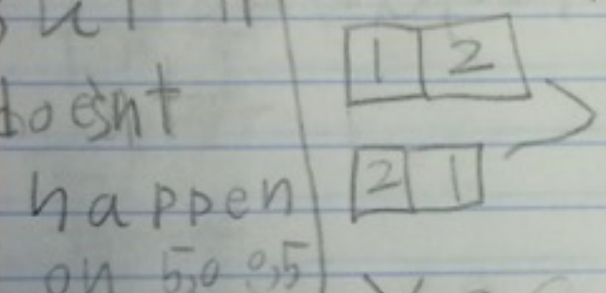
(Prove it)

9101123583145943707741561785981909987527965167303369549392 7291

This number bracelet is 62 #'s long and as big as it can get because all the bracelet starters in this bracelet makes a 60 # bracelet.

Does it matter
witch order the number
is in?

Example:



witch is
longer?

a: yes it does
matter

Does it happen
to other numbers?

a: yes it does
happen

(See exaple 2,1 1,2 9,4 4,9)

Q1:Does it matter witch order the number is in?

Example:
(1,2) or (2,1) Witch is longer?

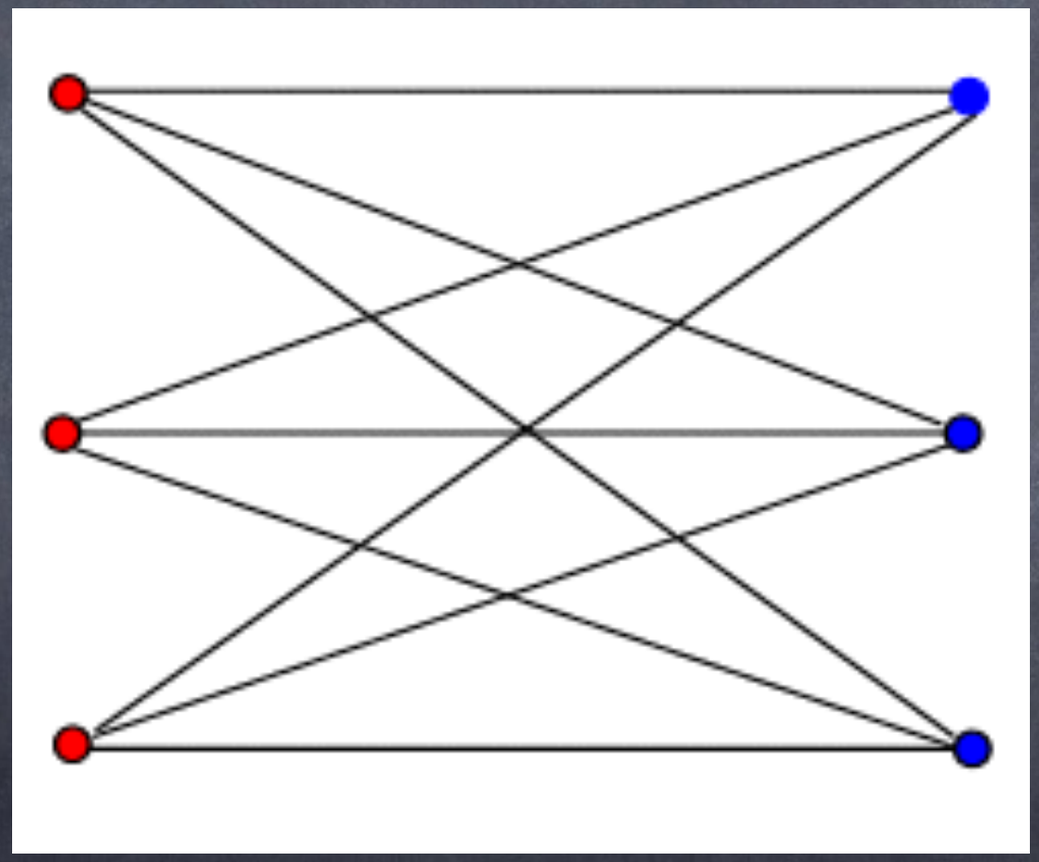
A: Yes it does matter

Q2: Does it happen to other numbers?

A2: Yes it does
(see exaple 2,1 1,2 9,4 4,9)

*But it doesnt happen on
5,0 0,5

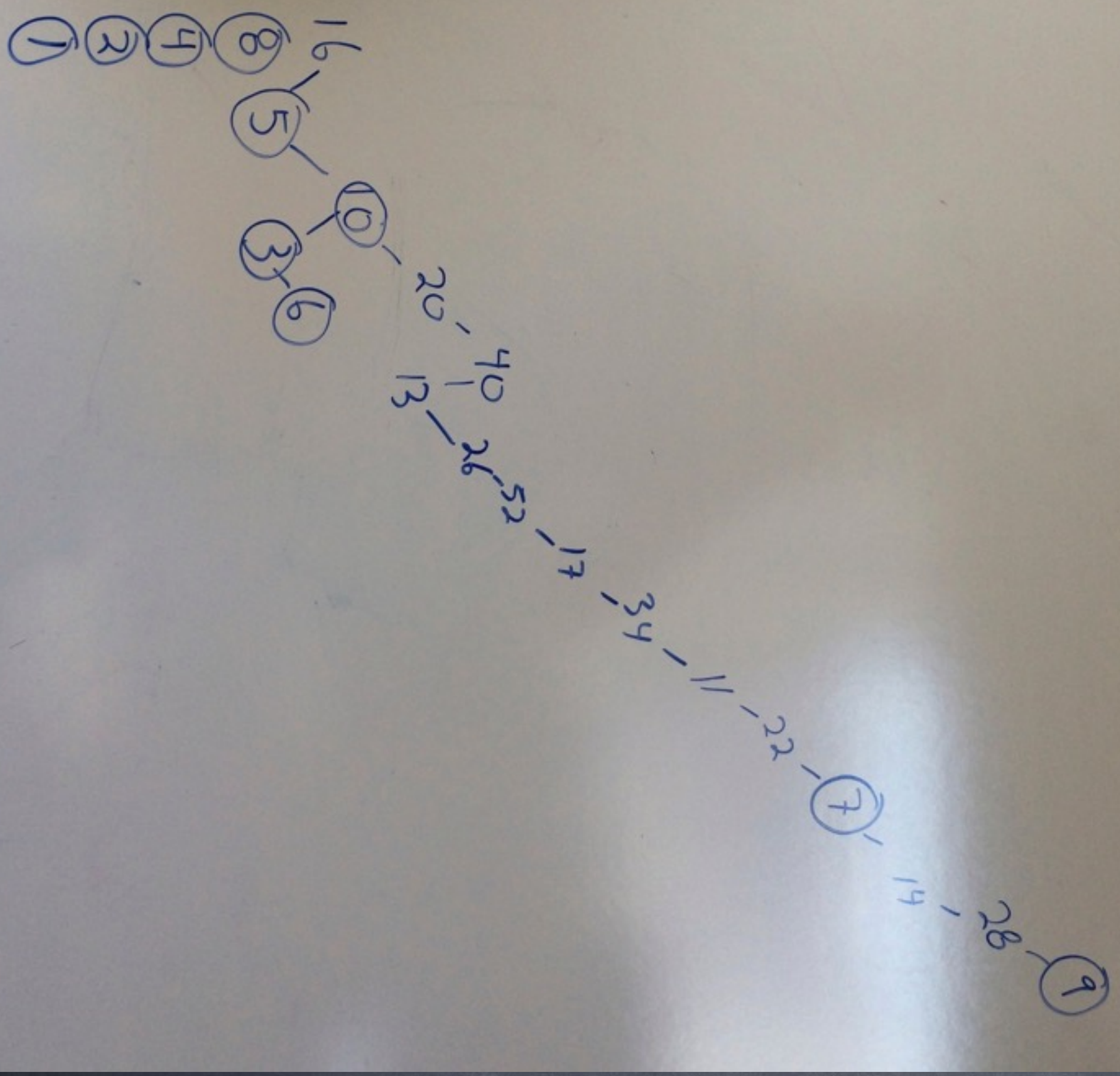
How many starting Pairs are there?



Collatz Conjecture(6-8)

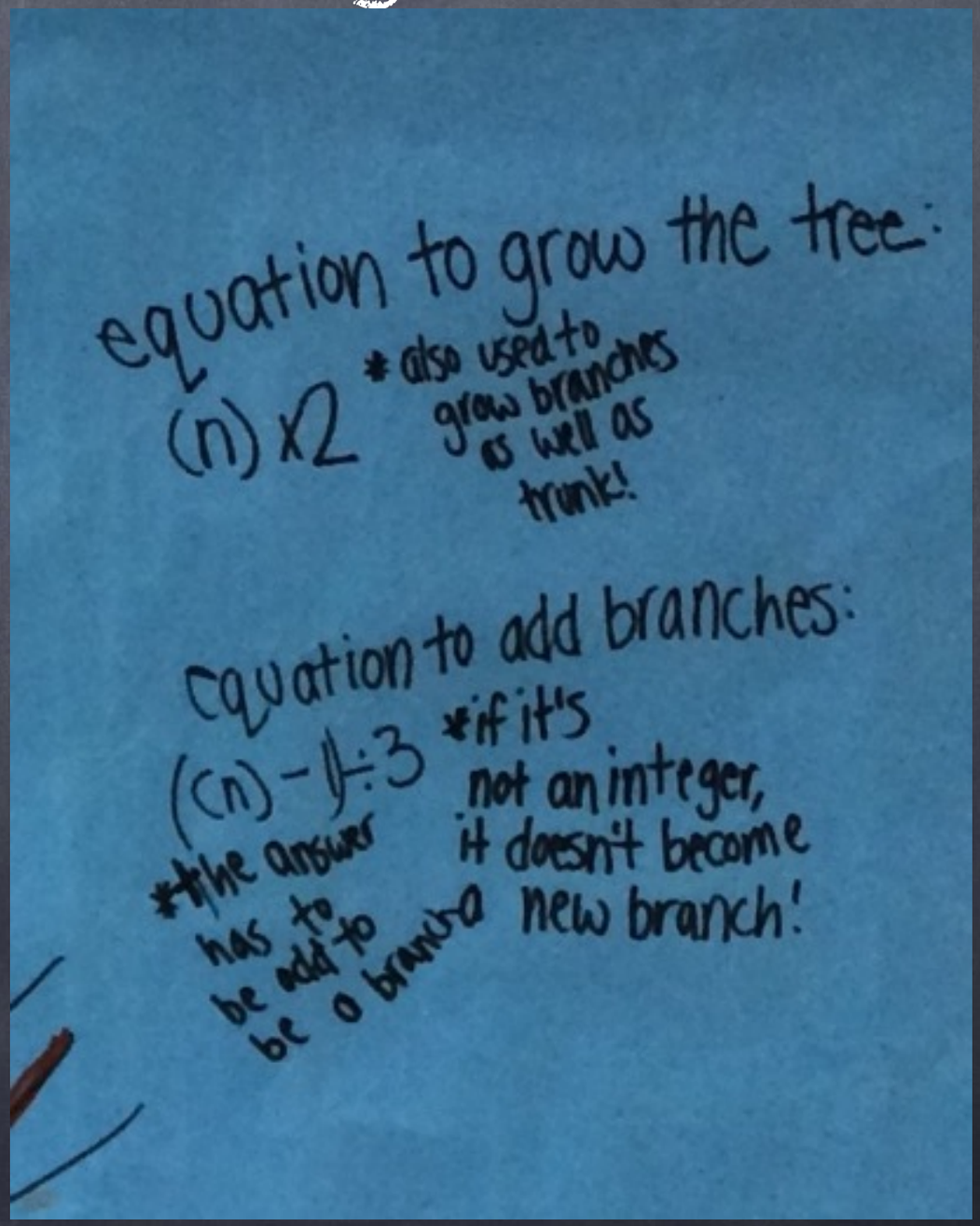
$$x_{n+1} = \begin{cases} x_n/2 & x_n \text{ even} \\ 3x_n + 1 & x_n \text{ odd} \end{cases}$$

- Roll a 10 sided die for a starting number, or choose a number between 1-10
- Apply the recursive formula to your starting number, recording each step
- The person who gets to 1 in the most number of steps wins

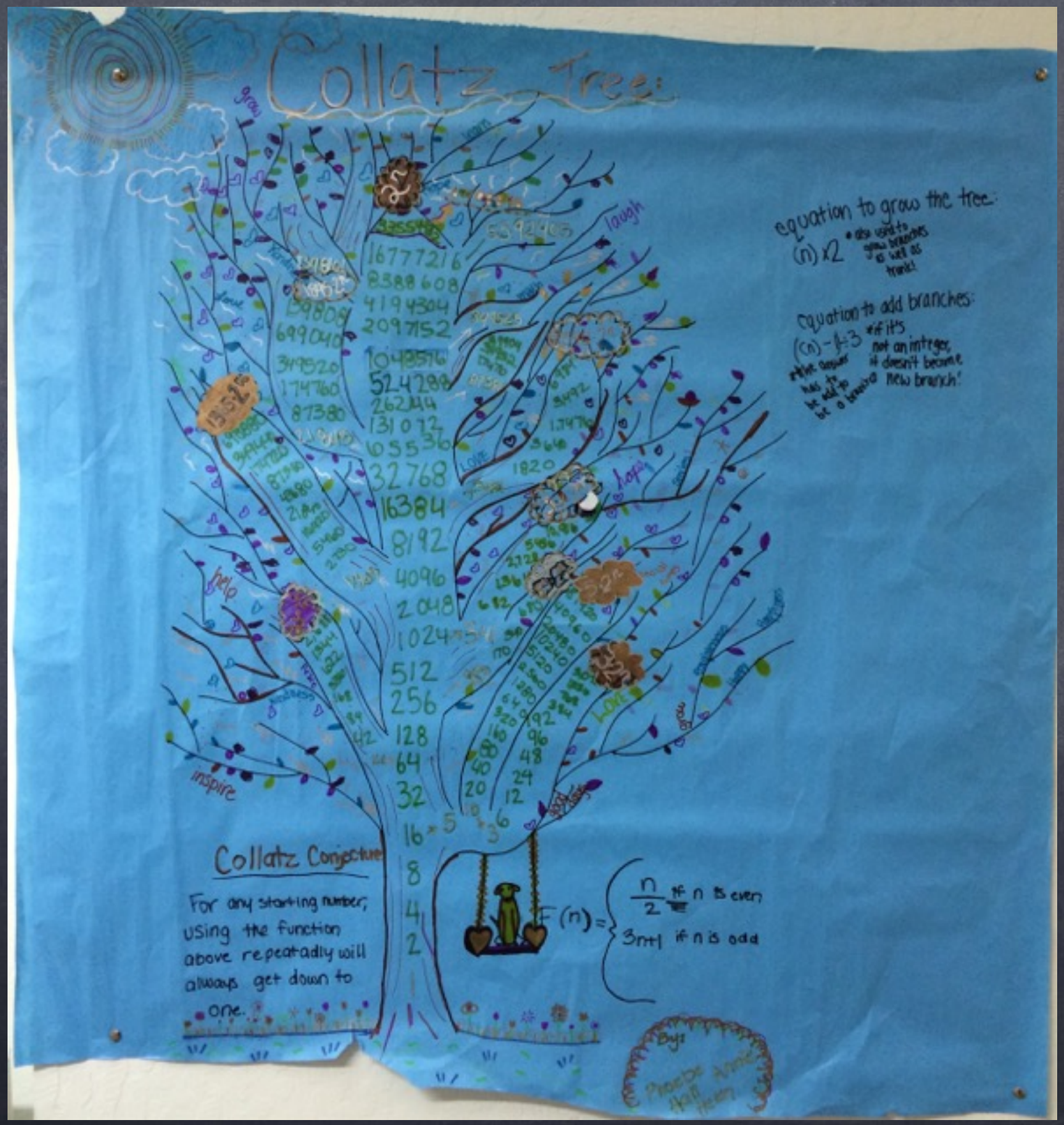

1
2
4
8
16
5
10
3
6
20
40
13
26
52
17
34
11
22
7
14
28
9

Can we reverse the process to grow the tree?

Doubling and Branching Rules



Collatz Tree



How do we fit this in?

- Squareable numbers: Area, Addition/subtraction
- Hundreds Game: Counting, Addition, Place Value/Hundreds Chart
- Subtraction Reversal Mysteries: Subtraction, Multiple of 9, Algebra
- Number Bracelets: Place Value, Addition, Modular Arithmetic, Combinations
- Collatz Tree: Equations and Expressions, Recursive Formulas, Divisibility Rules, Exponents, Prime Factorization

Key Take Aways

- Kids deserve time to play mathematically
- Celebrate the process, not perfection
- Exploration can overlap with skills practice

Resources

- www.artofmathstudio.wordpress.com
- www.mathpickle.com
- www.youcubed.com